STOPPING OF CHARGED PARTICLES IN A MAGNETIZED CLASSICAL PLASMA

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Abstract

The analytical and numerical investigations of the energy loss rate of the test particle in a magnetized electron plasma are developed on the basis of the Vlasov-Poisson equations, and the main results are presented. The Larmor rotation of a test particle in a magnetic field is taken into account. The analysis is based on the assumption that the energy variation of the test particle is much less than its kinetic energy. The obtained general expression for stopping power is analyzed for three cases: (i) the particle moves through a collisionless plasma in a strong homogeneous magnetic field; (ii) the fast particle moves through a magnetized collisionless plasma along the magnetic field; and (iii) the particle moves through a magnetized collisional plasma across a magnetic field. Calculations are carried out for the arbitrary test particle velocities in the first case, and for fast particles in the second and third cases. It is shown that the rate at which a fast test particle loses energy while moving across a magnetic field may be much higher than the loss in the case of motion through plasma without magnetic field.

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1 INTRODUCTION

Energy loss of fast charged particles in a plasma has been a topic of great interest since the 1950s [1-8] due to its considerable importance for the study of basic interactions of the charged particles in real media; moreover, recently it has also become a great concern in connection with heavy-ion driven inertial fusion research [5-8].

The nature of experimental plasma physics is such that experiments are usually performed in the presence of magnetic fields, and consequently it is of interest to investigate the effects of a magnetic field on the energy loss rate. Strong magnetic fields used in the laboratory investigations of plasmas can appreciably influence the processes determined by Coulomb collisions [9]. This influence is even more important in white dwarfs and in neutron stars, the magnetic fields on the surfaces of which can be as high as $10^5 - 10^{10}$ kG.

Stopping of charged particles in a magnetized plasma has been the subject of several papers [10-15]. Stopping of a fast test particle moving with velocity u much higher than the electron thermal velocity v_T was studied in Refs. [10,11,13]. Energy loss of a charged particle moving with arbitrary velocity was studied in Ref. [12]. The expression obtained there for the Coulomb logarithm, $L = \ln(\lambda_D/\rho_\perp)$ (where λ_D is the Debye length and $\rho_\perp$ is the impact parameter for scattering for an angle $\vartheta = \pi/2$), corresponds to the classical description of collisions. In the quantum-mechanical case, the Coulomb logarithm is $L = \ln(\lambda_D/\lambda_B)$, where λ_B is the de Broglie wavelength of plasma electrons [16].

In Ref. [15], the expressions were derived describing the stopping power of a charged particle in Maxwellian plasma placed in a classically strong (but not quantizing) magnetic field ($\lambda_B \ll a_c \ll \lambda_D$, where a_c is the electron Larmor radius), under the conditions when scattering must be described quantum mechanically. Calculations were carried out for slow test particles whose velocities satisfy the conditions $(m_e/m_i)^{1/3}v_T < u \ll v_T$, where m_i is the mass of the plasma ions and m_e is the electron mass.

The reaction of a uniform plasma to an electrostatic field of a moving test particle was studied by Rostoker and Rosenbluth [17] for two cases, in the presence or absence of a uniform magnetic field. For a test particle having velocity $u \gg v_T$ in the positive z direction, the dielectric function for no magnetic field gives a resonance $k_z = \omega_p/u$. As a result, the emission of plasma waves by the test particle with given k is concentrated on the cone forming an angle θ with respect to u, where $\cos\theta = k_z/k$. As shown by Rostoker and Rosenbluth, this leads to a Čherenkov-type shock front making the acute angle $\pi/2 - \theta$ with the negative z axis. Their treatment in the presence of a magnetic field was very general and involved no assumption concerning the relative magnitudes of ω_p and ω_c, i.e., the electron's plasma and cyclotron frequencies. Stopping power was not determined for any specific case. The authors were aware that in the case when $\omega_c \gg \omega_p$, where field electrons in the lowest order can respond only to the waves in the direction of $\mathbf{B}_0$, the resonance caused by the dielectric function has a different form, with $k = \omega_p/u$, being independent of the $\mathbf{k}$ direction.

The electrostatic field of a moving test particle in such magnetized plasma was studied by Ware and Wiley [18].

In the present paper we calculate, in a framework of dielectric theory, the energy loss rate of a test particle moving in magnetized plasma. We consider the test particle interaction only with the electron component of plasma, since it is this interaction that dominates the stopping of a test particle [19,20]. Besides, in contrast with the papers [10-15], Larmor rotation of a test particle in a magnetic field is taken into account.

In Sec. II, linearized Vlasov-Poisson equations are solved by means of Fourier analysis in order to obtain a general form for the linearized potential generated in a magnetized Maxwellian plasma by a test particle and for the energy loss rate of the test particle.

In Sec. III, the energy loss in a Maxwellian collisionless plasma in the presence of a strong magnetic field is examined. Calculations are carried out for arbitrary test particle velocities. In this case, plasma oscillations are also excited, though their spectra in the strong magnetic field differ from normal ones.

In Sec. IV, the energy loss rate in a cold plasma is calculated in the case when the fast particle moves along ($\vartheta = 0$) and across ($\vartheta = \pi/2$) the magnetic field. It is shown that in the first case, the energy loss rate is less than Bohr's result. In the second case, the energy loss rate can be much higher than Bohr's result.

In Sec. V, we present a qualitative discussion of obtained results. In the Appendix, analysis of the function $Q_\nu(z)$ is given.

2 BASIC RELATIONS

We consider a nonrelativistic charged particle having charge Ze that moves in a magnetized plasma at an angle ϑ with respect to the magnetic field directed along the z axis. We assume that the energy variation of the particle is much smaller than its kinetic energy. In this case the charge density associated with the test particle is given by the following expression:

$$\rho_0(\mathbf{r}, t) = Ze\delta(x - a\sin(\Omega_c t))\delta(y - a\cos(\Omega_c t))\delta(z - u_0 t), \tag{1}$$

where u_0 and v are the particle velocity components along and across from the magnetic field $\mathbf{B}_0$ ($u_0 = u\cos\vartheta$, $v = u\sin\vartheta$), where u is the particle velocity, $\Omega_c = ZeB_0/Mc$, $a = v/\Omega_c$, and M are the Larmor frequency, the Larmor radius, and the mass of the particle, respectively, and c is the speed of light.

The linearized Vlasov equation of the plasma may be written as

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\frac{\partial f_1}{\partial \mathbf{r}} + \omega_c\left[\mathbf{v} \times \mathbf{b}_0\right]\frac{\partial f_1}{\partial \mathbf{v}} = \frac{e}{m}\frac{\partial \varphi}{\partial \mathbf{r}}\frac{\partial f_0}{\partial \mathbf{v}}, \tag{2}$$

where the self-consistent electrostatic potential φ is determined by Poisson's equation

$$\nabla^2\varphi = -4\pi\rho_0(\mathbf{r}, t) - 4\pi e\int d\mathbf{v} f_1(\mathbf{r}, \mathbf{v}, t), \tag{3}$$

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where $\mathbf{b}_0$ is the unit vector parallel to $\mathbf{B}_0$, e, m, and ω_c are the charge, mass, and Larmor frequency of plasma electrons, respectively, f_0 is the unperturbed distribution function of plasma electrons, which is taken uniform and Maxwellian,

$$f_0(v) = \frac{n_0}{(2\pi v_T^2)^{3/2}} \exp\left(-\frac{v^2}{2v_T^2}\right), \tag{4}$$

with $v_T = \sqrt{k_B T/m}$. Here, n_0 is the unperturbed number density of the plasma electrons.

By solving Eqs. (2) and (3) in space-time Fourier components, we obtain the following expression for the electrostatic potential:

$$
\begin{aligned}
\varphi(\mathbf{r}, t) &= \frac{Ze}{\pi} \sum_{n=-\infty}^{\infty} \exp[-in(\psi + \Omega_c t)] \int_0^\infty dk_\perp k_\perp J_n(k_\perp a) J_n(k_\perp \rho) \\
&\times \int_{-\infty}^{+\infty} \frac{dk_z \exp(ik_z \xi)}{\left(k_z^2 + k_\perp^2\right) \varepsilon(k_z, k_\perp, k_z u_0 + n\Omega_c)},
\end{aligned}
\tag{5}
$$

where $k^2 = k_z^2 + k_\perp^2$, $\xi = z - u_0 t$, J_n is the nth order Bessel function, ρ, ψ, and z are the cylindrical coordinates of the observation point, and $\varepsilon(k_z, k_\perp, \omega)$ is the plasma dielectric function, which has been given by many authors [21,22], and may be written in the form

$$\varepsilon(k_z, k_\perp, \omega) = 1 + \frac{1}{k^2 \lambda_D^2} \left[1 + 2ip \int_0^\infty dt \exp\left(2ipt - W\right)\right] \tag{6}$$

with $p = \omega/\sqrt{2}k v_T$ and

$$W = t^2 \cos^2 \alpha + k^2 a_c^2 \sin^2 \alpha \left[1 - \cos\left(\frac{\sqrt{2}\omega_c t}{k v_T}\right)\right]. \tag{7}$$

Here, α is the angle between the wave vector $\mathbf{k}$ and the magnetic field.

The result represents a dynamical response of the medium to the motion of the test particle in the presence of the external magnetic field; it takes the form of an expansion over all the harmonics of the Larmor frequency of the particle.

The energy loss rate (ELR) S of a fast charge is defined as the energy loss of the charge in a unit time due to interactions with the plasma electrons. From Eq. (5) it is straightforward to calculate the electric field $\mathbf{E}(\mathbf{r}, t) = -\boldsymbol{\nabla}\varphi(\mathbf{r}, t)$, and the stopping force acting on the particle. Then, the ELR of the test particle becomes

$$
\begin{aligned}
S &= \frac{2Z^2 e^2}{\pi} \sum_{n=-\infty}^{\infty} \int_0^{k_{\max}} dk_\perp k_\perp J_n^2(k_\perp a) \\
&\times \int_0^\infty dk_z \frac{k_z u_0 + n\Omega_c}{k_z^2 + k_\perp^2} \mathrm{Im} \frac{-1}{\varepsilon(k_z, k_\perp, k_z u_0 + n\Omega_c)},
\end{aligned}
\tag{8}
$$

with $k_{\max} = 1/r_{\min}$, where $r_{\min}$ is the effective minimum impact parameter. Here $k_{\max}$ has been introduced to avoid the divergence of the integrals caused by the incorrect treatment of the short-range interaction between the test particle and the plasma electrons within the linearized Vlasov theory.

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3 ELR IN PLASMA IN THE PRESENCE OF A STRONG MAGNETIC FIELD

Let us analyze expression (8) in the case when a particle moves in a plasma with a sufficiently strong magnetic field. Let us assume the magnetic field, on one hand, reasonably weak and not to be quantized ($\hbar\omega_c < k_B T$ or $a_c \gg \lambda_B$), and, on the other hand, comparatively strong so that the cyclotron frequency of the plasma electrons exceeds the plasma frequency $\omega_p = \sqrt{4\pi n_0 e^2/m}$ or $a_c \ll \lambda_D$, where a_c is the Larmor radius and λ_D is the Debye length $\lambda_D = v_T/\omega_p$. Because of this assumption, the perpendicular cyclotron motion of the test and plasma particles is neglected. The test particle's velocity parallel to $\mathbf{B}_0$ is taken as u_0. The generation of an electrostatic wake by a superthermal test electron in a magnetized electron plasma in this limit has been discussed by Ware and Wiley [18].

In the limit of sufficiently strong magnetic field, Eq. (8) becomes

$$S = \frac{2Z^2 e^2 u_0}{\pi} \int_0^{k_{\max}} dk_\perp k_\perp \int_0^\infty dk_z \frac{k_z}{k^2} \text{Im} \frac{-1}{\varepsilon_\infty(k_z, k_\perp, k_z u_0)} \tag{9}$$

with

$$\varepsilon_\infty(k_z, k_\perp, \omega) = 1 + \frac{1}{k^2 \lambda_D^2} \left[X\left(\frac{\omega}{k_z v_T}\right) + i\frac{|k_z|}{k_z} Y\left(\frac{\omega}{k_z v_T}\right) \right], \tag{10}$$

where $W(z) = X(z) + iY(z)$ is the plasma dispersion function [23],

$$X(z) = 1 - \sqrt{2}z Di\left(\frac{z}{\sqrt{2}}\right), \tag{11}$$

$$Y(z) = \sqrt{\frac{\pi}{2}} z \exp\left(-\frac{z^2}{2}\right), \tag{12}$$

$$Di(z) = \exp\left(-z^2\right) \int_0^z dt \exp\left(t^2\right) \tag{13}$$

is the Dawson integral [23]. At large values of its argument, the Dawson integral has the value $Di(z) \simeq 1/2z + 1/4z^3$.

Substituting Eq. (10) into Eq. (9) and making the substitutions $\lambda = u_0/v_T$, $B = k_{\max}\lambda_D$ we obtain

$$\begin{aligned} S &= \frac{Z^2 e^2 v_T}{2\pi \lambda_D^2} \lambda \left\{ \frac{Y(\lambda)}{2} \ln \frac{Y^2(\lambda) + (B^2 + X(\lambda))^2}{Y^2(\lambda) + X^2(\lambda)} \right. \\ &\quad + B^2 \left[\frac{\pi}{2} - \arctan \frac{B^2 + X(\lambda)}{Y(\lambda)} \right] \\ &\quad \left. + X(\lambda) \left[\arctan \frac{X(\lambda)}{Y(\lambda)} - \arctan \frac{B^2 + X(\lambda)}{Y(\lambda)} \right] \right\}. \end{aligned} \tag{14}$$

The maximum value of $k_\perp$, $k_{\max}$, will be a_c^{-1} for fusion plasmas, since the magnetized plasma approximation that neglects the perpendicular motion of electrons ceases to be valid for collision parameters less than a_c.

The first term of Eq. (14) is a contribution to the frictional drag due to collisions with the plasma electrons. It is incomplete because the analysis treats the background electrons as a continuous fluid and there is no allowance being made for the recoil of the test particle due to each collision. The other terms are associated with the resonance giving rise to plasma wave emission.

From Eqs. (9)-(14) we can assume that the main contribution in the ELR is given by the values of the particle's velocity, for which $X(\lambda) < 0$ and $Y(\lambda) \ll |X(\lambda)|$. These conditions correspond to excitation of plasma waves by a moving particle. As shown by Rostoker and Rosenbluth [17], the plasma waves were not determined for any specific case. They were aware that for the case $\omega_c \gg \omega_p$, where the plasma electrons in the lowest order can respond to the waves only in the direction of $\mathbf{B}_0$, the resonance caused by the dielectric function has a different form $k = \omega_p/u_0$, being independent of the $\mathbf{k}$ direction. Plasma waves involved in this case are oblique plasma waves having the approximate dispersion relation $\omega_k = \omega_p k_z/k$. In Secs. III A and III B the expression (14) is evaluated for large and small test particle velocities.

3.1 ELR for small velocities

When a test particle moves slowly through a plasma, the electrons have much time to experience the particle's attractive potential. They are accelerated towards the particle, but when they reach its trajectory the particle has already moved forward a little bit. Hence, we expect an increased density of electrons at some place in the trail of the particle. This negative charge density pulls back the positive particle and gives rise to the ELR.

The Taylor expansion of Eq. (14) for small u_0 ($\lambda \ll 1$) yields the "friction law"

$$S = \frac{Z^2 e^2 v_T}{2\sqrt{2\pi}\lambda_D^2} \left[\lambda^2 \mathcal{R}_1 - \lambda^4 \mathcal{R}_2 + O(\lambda^6) \right] \tag{15}$$

with the "friction coefficient"

$$\mathcal{R}_1 = \ln\left(1 + B^2\right) \tag{16}$$

and the λ^4 coefficient

$$\mathcal{R}_2 = \frac{1}{2} \ln\left(1 + B^2\right) - \frac{1}{2}\left(1 - \frac{\pi}{6}\right) - \frac{\pi}{4}\frac{1}{(1 + B^2)^2} + \frac{\pi}{6}\frac{1}{(1 + B^2)^3}. \tag{17}$$

Note that $B = \omega_c/\omega_p$ and therefore $B \gg 1$. The Coulomb logarithms in Eqs. (16) and (17) are then the leading terms. We obtain

$$S = \frac{Z^2 e^2 v_T}{2\sqrt{2\pi}\lambda_D^2} \left\{ 2\lambda^2 \ln B - \lambda^4 \left[\ln B - \frac{1}{2}\left(1 - \frac{\pi}{6}\right) \right] + O(\lambda^6) \right\}. \tag{18}$$

The most important property of the ELR at small velocities is $S \propto u_0^2$ provided that the density is not too high ($\omega_p < \omega_c$). This looks like the friction law of a viscous fluid, and accordingly $\mathcal{R}_1$ is called the friction coefficient.

However, in the case of an ideal plasma it should be noted that this law does not depend on the plasma viscosity and is not a consequence of electron-electron collisions with small impact parameter. These collisions are neglected in the Vlasov equation. As described above, it is rather the fact that the dressing of the test particle takes some time and produces the negative charge behind the particle leading to the drag.

3.2 ELR for large velocities

For large u_0 ($u_0 \gg v_T$) we have $X(\lambda) \simeq -1/\lambda^2$, $Y(\lambda) \simeq 0$. In this case Eq. (14) becomes

$$S \simeq \frac{Z^2 e^2 \omega_p^2}{2 v_T} \frac{v_T}{u_0}. \tag{19}$$

From Eq. (19) we can assume that the ELR is $2L = 2\ln(k_{\max}^{(0)} u_0/\omega_p)$ (where $k_{\max}^{(0)}$ is a cutoff parameter in a plasma in the absence of magnetic field) times smaller than the Bohr ELR [24].

Our assumption made at the beginning of this section was that the classical approach in consideration of energy losses in plasma placed in a strong magnetic field limits the values of the magnetic field itself and values of temperature and plasma concentrations. From these conditions we can obtain

$$3 \times 10^{-6} n_0^{1/2} < B_0 < 10^5 T, \tag{20}$$

where n_0 is measured in cm^{-3}, T is measured in eV, and B_0 in kG. Conditions (20) are always true in the range of parameters $n_0 < 10^{15}$ cm^{-3}, $B_0 < 100$ kG, $T > 10^{-3}$ eV.

In Fig. 1, the ELR is plotted as a function of parameter λ for $T = 10$ eV, $n_0 = 10^{14}$ cm^{-3}, and for two different values of B_0: $B_0 = 50$ kG (dotted line) and $B_0 = 80$ kG (solid line). The peak corresponds to excitation of plasma waves by a moving particle.

4   ELR OF A FAST CHARGED PARTICLE IN COLD MAGNETIZED PLASMA

We shall further analyze Eq. (8) in the case when the fast particle moves in a cold plasma whose longitudinal dielectric function is given by the following expressions [25,26]:

$$\varepsilon(k_z, k_\perp, \omega) = \varepsilon(\omega) \cos^2 \alpha + h(\omega) \sin^2 \alpha \tag{21}$$

with

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}, \quad h(\omega) = 1 + \frac{\omega_p^2(\omega + i\nu)}{\omega[\omega_c^2 - (\omega + i\nu)^2]}, \tag{22}$$

where ν is the effective collision frequency. The collisions are negligible if the frequency of collisions with large scattering angle between the electrons is small

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compared with the plasma frequency ω_p. The cross section for collisions with scattering angles of $90°$ or more is $\sigma_{90°} = \pi r_{90°}^2 = \pi \left(e^2/k_B T\right)^2$ and the frequency of such collisions $\nu = n_0 \sigma_{90°} v_T$. Thus

$$\frac{\nu}{\omega_p} = \frac{1}{4}\left[\frac{\pi}{2}n_0\left(\frac{e^2}{k_B T}\right)^3\right]^{1/2}. \tag{23}$$

If $T \gg 6.6 \times 10^{-8} n_0^{1/3}$, then $\nu \ll \omega_p$ and the collisions in the plasma may be ignored.

In Eq. (8) we introduced a cutoff parameter k_{max} in order to avoid the logarithmic divergence at large $k_\perp$. This divergence corresponds to the incapability of the linearized Vlasov theory to treat close encounters between the test particle and the plasma electrons properly. The full nonlinear Vlasov equation accurately describes the scattering of individual electrons with the test particle in accordance with the Rutherford scattering theory. The exact expression for energy transfer in the Rutherford two-body collision is

$$\Delta E(\rho) = \frac{(\Delta \mathbf{p})^2}{2m} = \frac{2Z^2 e^4}{m v_r^2}\frac{1}{\left(\frac{Ze^2}{m v_r^2}\right)^2 + \rho^2}, \tag{24}$$

where $v_r \simeq (u^2 + v_T^2)^{1/2}$ is the mean relative velocity between the test particle and the electron. From the denominator in Eq. (24) it follows that the effective minimum impact parameter is $r_{min} = Ze^2/m v_r^2$, which is often called the "distance of closest approach." Thus,

$$k_{max} = \frac{1}{r_{min}} = \frac{m\left(u^2 + v_T^2\right)}{Ze^2} \tag{25}$$

ensures agreement of Eq. (8) with the Rutherford theory for small impact parameters. When $u > 2Ze^2/\hbar$, the de Broglie wavelength begins to exceed the classical distance of closest approach. Under these circumstances we choose $k_{max} = 2mu/\hbar$.

4.1 Longitudinal motion of a particle ($\vartheta = 0$)

In the case of an incidence angle $\vartheta = 0$ of the test particle, we obtain from Eqs. (8) and (21) the following expression:

$$S = \frac{2Z^2 e^2}{\pi u_0}\int_0^{k_{max}} dk_\perp k_\perp \int_0^\infty d\omega\, \omega \text{Im}\frac{-1}{k_\perp^2 h(\omega) + \left(\omega^2/u_0^2\right)\varepsilon(\omega)}. \tag{26}$$

Due to the resonant character of the integral over ω in the expression (26), the main contribution to the energy losses gives those ranges of integration where $\text{Im}\varepsilon \ll \text{Re}\varepsilon$ and $\text{Im}h \ll \text{Re}h$. These conditions are true when $\nu \ll \omega_p$. By using the property of the Dirac δ function from expression (26), we have

$$S = \frac{2Z^2 e^2}{u_0}\int_0^{k_{max}} dk_\perp k_\perp \int_0^\infty d\omega\, \omega \delta\left[k_\perp^2 h(\omega) + \left(\omega^2/u_0^2\right)\varepsilon(\omega)\right]. \tag{27}$$

In the expression (27) the argument of the δ function defines the frequencies of normal oscillations of a magnetized plasma in the long-wavelength approximation. In general, they are studied in Refs. [27,28] in more detail for electron plasma. After integration in expression (27), we have

$$S = \frac{Z^2 e^2}{u_0} \int_C \frac{d\omega \omega}{|h(\omega)|}, \tag{28}$$

where the range of integration C can be determined from the inequality $P(\omega) < -\omega^2/k_{\max}^2 u_0^2$ and $P(\omega) = h(\omega)/\varepsilon(\omega)$.

Integrating over frequency in the expression (28), we obtain finally

$$S = \frac{Z^2 e^2 v_T}{4\lambda\lambda_D^2} \left[F(\beta) - \sqrt{F^2(\beta) - 4\beta^2} + 2\ln \frac{F(\beta) + \sqrt{F^2(\beta) - 4\beta^2}}{2(1+\beta^2)} \right], \tag{29}$$

where $\beta = \omega_c/\omega_p$, $\lambda = u_0/v_T$, and $F(\beta) = 1 + \beta^2 + \lambda^2 B^2$, with

$$B = k_{\max}\lambda_D = \begin{cases} (k_B T/Ze^2 \lambda_D^{-1})\lambda^2, & 1 \ll \lambda < 2Ze^2/\hbar v_T, \\ (2k_B T/\hbar\omega_p)\lambda, & \lambda > 2Ze^2/\hbar v_T. \end{cases} \tag{30}$$

As it follows from the expression (29), for low-intensity magnetic fields ($\beta < 1$), the ELR tends to the well-known Bohr result [24]

$$S_B = \frac{Z^2 e^2 \omega_p^2}{u_0} \ln \left(\frac{k_{\max} u_0}{\omega_p} \right). \tag{31}$$

Meanwhile, for the high-intensity magnetic fields ($\beta > 1$), the expression (29) tends to a constant value $q^2\omega_p^2/2u_0$, which also follows from Eq. (14) when thermal motion of electrons is ignored. For arbitrary values of β, the ELR do not exceed the Bohr losses (see Fig. 2).

4.2 Transversal motion of a particle ($\vartheta = \pi/2$)

In the case of the transversal motion of a particle, $u_0 = 0$, and the general expression (8) becomes

$$S = \frac{2Z^2 e^2 \Omega_c^2}{\pi v} \sum_{n=1}^{\infty} n Q_n(s) \operatorname{Im} \left[\frac{-1}{\varepsilon(n\Omega_c) T(n\Omega_c)} \right], \tag{32}$$

where $s = k_{\max} a$,

$$T(\omega) = \sqrt{\frac{|P(\omega)| + \operatorname{Re}P(\omega)}{2}} + i\operatorname{sgn}\left[\operatorname{Im}P(\omega)\right] \sqrt{\frac{|P(\omega)| - \operatorname{Re}P(\omega)}{2}}, \tag{33}$$

$$Q_\nu(s) = \pi \int_0^s dx J_\nu^2(x). \tag{34}$$

Function $Q_\nu(s)$ is examined in the Appendix, where asymptotic values are also given. The function $Q_\nu(s)$ is shown to be exponentially small at $\nu > s$. Therefore, the series entering Eq. (32) is cut at $n_{\max} \simeq s$ and the ELR is determined by harmonics having $n < n_{\max}$.

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Let us study Eq. (32) in the range of strong magnetic fields. Two cases must be mentioned here.

(i) $c = \omega_c/\Omega_c$ is a fraction. In this case, from Eq. (32) we find

$$S \simeq \frac{Z^2 e^2 \omega_p^2}{\pi v} \frac{\nu}{\Omega_c} \sum_{n=1}^{\infty} \frac{1}{n^2} Q_n(s) \left[1 + \frac{n^4}{(n^2 - c^2)^2}\right]. \tag{35}$$

From Eq. (35) it follows that the energy loss decreases inversely proportional to the magnetic field.

(ii) $c = 1$ (electron test particle). From Eq. (32) in this case we find

$$S \simeq \frac{Z^2 e^2 \omega_p^2}{\pi v} \frac{\Omega_c}{\nu} Q_1(s). \tag{36}$$

In this case the ELR increases proportionally to the magnetic field.

The above examples of the asymptotic ELR dependence on the value of the magnetic field show strong dependence of ELR on the mass of a test particle in the case when the magnetic field is sufficiently strong.

From Eq. (32) it is easy to trace qualitatively the behavior of energy losses as a function of magnetic field in the general case. Thus, as it follows from Eq. (32), the ELR is maximal for those values of the magnetic field for which $\varepsilon(n\Omega_c)$ is small. The smallness $\varepsilon(n\Omega_c)$ means that the dependence of the ELR from the magnetic field reveals maxima at integer values of parameter $b = a/\lambda_p \equiv \omega_p/\Omega_c$, where $\lambda_p = 2\pi v/\omega_p$ is the plasma oscillations' wavelength.

Figure 3 shows ELR to Bohr ELR ratio as a function of parameter b in two cases: for proton (dotted line) and electron (solid line) test particle. The plasma and/or particle parameters are taken equal to $T = 100$ eV, $n_0 = 10^{18}$ cm^{-3}, $v/\omega_{pe} = 0.01$, and $\lambda = 10$. As it follows from Fig. 3, ELR oscillates as a function of magnetic field and many times exceeds the usual Bohr ELR.

5   SUMMARY

The purpose of this work was to analyze the energy loss rate (ELR) of a charged particle in a magnetized classical plasma. Larmor rotation of a test particle in a magnetic field was taken into account. A general expression obtained for ELR was analyzed in three particular cases: in a Maxwellian plasma under a strong magnetic field; in a cold plasma when the particle moves along the magnetic field; and in a cold plasma when the particle moves across the magnetic field.

The energy loss in a Maxwellian plasma, both in the presence of a strong magnetic field and in its absence, is conditioned by the induced plasma waves. In the presence of a strong magnetic field, the dispersion of plasma oscillations is perceptibly altered. From the expression (10) one may see that the frequency and the damping rate of these waves depend on the direction of spreading relative to the magnetic field. The maximal frequency of these waves is reached when they are spread along the magnetic field. Across the magnetic field, they cannot be spread. It can be noticed that for the electron plasma oscillations, these effects are analyzed in detail in Refs. [17,27,28].

From the results obtained in Sec. IV, one may conclude that the ELR essentially depends on the particle's incident angle with respect to magnetic field. In the case of longitudinal motion ($\vartheta = 0$), the ELR is less than or comparable with Bohr's result, and in the limit of strong magnetic fields, ELR depends only on the density of the plasma. When the particle moves across the magnetic field ($\vartheta = \pi/2$), the latter essentially affects the ELR value. First, ELR has an oscillatory character of dependence on a magnetic field, becoming maximal at integer values of parameter $b = \omega_p/\Omega_c$ (the ratio of Larmor circle length and plasma wave wavelength). Second, ELR in the magnetized plasma at $\vartheta = \pi/2$ is much greater than the Bohr result. Third, the strong dependence of ELR on the mass of the test particle can be seen when the magnetic field is sufficiently strong. If thermal motion of plasma electrons is considered, the results obtained in Sec. IV will be preserved in general. However, the new effects related to the increased number of normal plasma modes will originate. In particular, at $\vartheta = \pi/2$, the new mechanism of stopping could be expected, namely stopping by excitation of the Bernstein oscillations [21].

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APPENDIX

Let us examine the properties of function $Q_\nu(s)$ determined by Eq. (34). To find the asymptotic value of that function at $s \gg 1$ and $s > \nu$, we partition the area of integration in Eq. (34) into areas $x < \nu$ and $\nu < x < s$ and use the asymptotic presentation of the Bessel function at $x > \nu$ [29]. Thus, we find

$$Q_\nu(s) \simeq q_\nu + \ln\frac{s}{\nu} + \cos(\pi\nu)\left[\text{si}(2s) - \text{si}(2\nu)\right] - \sin(\pi\nu)\left[\text{ci}(2s) - \text{ci}(2\nu)\right], \quad \text{(A1)}$$

where $\text{si}(z)$ and $\text{ci}(z)$ are integral sine and cosine, respectively,

$$q_\nu = \pi \int_0^\nu dx\, J_\nu^2(x). \quad \text{(A2)}$$

Numbers q_ν are less than 1, and slowly fall off as the ν increases. Here we point out some values of q_ν: $q_1 \simeq 0.225$, $q_{20} \simeq 0.096$, $q_{100} \simeq 0.057$.

At $s < \nu$, the argument of the Bessel function is lower than the index. In this case, the Bessel function is exponentially small, and at a fixed value of s, $Q_\nu(s)$ exponentially vanishes as ν increases.

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Figure Captions

Fig.1. ELR (in MeV/sec) of a proton as a function of the dimensionless parameter $\lambda = u_0/v_T$ in the case when the particle moves in Maxwellian plasma ($T = 10$ eV, $n_0 = 10^{14}$ cm^{-3}) placed in a strong magnetic field for two values of B_0: $B_0 = 50$ kG (dotted line) and $B_0 = 80$ kG (solid line).

Fig.2. Dependence of function $R = S/S_B$ on the dimensionless magnetic field $\beta = \omega_c/\omega_p$ in the case when the particle moves along the magnetic field for the values of parameter $\lambda = 5$ (dotted line) and $\lambda = 10$ (solid line). Plasma parameters are taken equal to $T = 100$ eV and $n_0 = 10^{22}$ cm^{-3}, while $Z = 1$ for the test particle.

Fig.3. Dependence of a function $R = S/S_B$ on the dimensionless parameter $b = \omega_p/\Omega_c$ for proton (dotted line) and electron (solid line). Parameters are taken equal to $T = 100$ eV, $n_0 = 10^{18}$ cm^{-3}, $\lambda = 10$, and $\nu/\omega_p = 0.01$.